Test 1 / Numerical Mathematics 1 / May 3d 2021, University of Groningen

Instructions

- This examination is open book.
- **You have 1 hour and 45 minutes to finalize the test and upload your solutions. People with special needs (according to the official information of the Educational support center) have 2 hours in total.**
- The grade will be computed as the number of obtained points, plus 1.
- Do not communicate with other students during the examination.
- Sign the pledge and upload it as a separate file in the same place where you will upload your answers.
- Stay in the online main collaborate room during the examination. Information delivered there is official and part of the instructions of the examination. The activity in the room will be recorded.
- The examination must be written by hand **in a tidy and legible way**, scanned and uploaded as PDF to Nestor. Of course you can also use a tablet to write your answers.
- **Upload the PDF in vertical orientation, such that it requires no rotation to be readable.**
- All answers need to be justified using mathematical arguments.
- Oral checks may be run afterwards, either randomly and/or in case of suspicion of fraud.
- **If you do not follow these instructions you will receive the minimal grade.**

Questions

Consider an arbitrary scalar function $g \in C^5([a,b])$, and the approximating polynomial $z(x)$ defined as:

$$z(x) = g(c) + g'(c)(x-c) + \frac{1}{2}g''(c)(x-c)^2 + \frac{1}{6}g'''(c)(x-c)^3$$

with $c = (a+b)/2$.

(a) $\boxed{2}$ Compute an error bound ϵ (independent on x) such that $|g(x) - z(x)| \leq \epsilon$, $\forall x \in [a,b]$, where a, b and g appear explicitly in ϵ. Verify that ϵ goes to zero when $b \to a$.

(b) $\boxed{2}$ Use the approximating polynomial $z(x)$ to define a numerical integration rule of $\tilde{I}(g)$ over $[a,b]$. Complicated non-zero integrals can be just left uncalculated.

(c) $\boxed{1.5}$ What is the degree of exactness of the resulting integration method? Justify rigorously your answer.

(d) $\boxed{1}$ Determine an error bound for $|I(g) - \tilde{I}(g)|$ where g, b and a appear explicitely.

(e) $\boxed{1.5}$ Derive a composite integration rule on the grid $x_i = a + ih$, with $i = 0, ..., N$ and $N = (b-a)/h$ using the quadrature defined above $\tilde{I}(g)$.

(f) $\boxed{1}$ Prove that the composite rule of Question (e) allows to the numerical integration error to go to zero when $h \to 0$. For this you may need to calculate some integral.